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Sensitivity Analysis of Dynamic Tariff Method for Congestion Management in Distribution Networks

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Abstract—**The dynamic tariff (DT) method is designed for the distribution system operator (DSO) to alleviate the congestions that might occur in a distribution network with high penetration of distribute energy resources (DERs). Sensitivity analysis of the DT method is crucial because of its decentralized control manner. The sensitivity analysis can obtain the changes of the optimal energy planning and thereby the line loading profiles over the infinitely small changes of parameters by differentiating the KKT conditions of the convex quadratic programming, over which the DT method is formed. Three case studies were conducted to demonstrate the impact of small and big changes of parameters on the line loading profiles and the effectiveness of the DT method.**

Index Terms--**Congestion management, convex quadratic programming, distribution system operator (DSO), distribute energy resources (DERs), sensitivity analysis.**

I. INTRODUCTION

As the penetration level of distributed energy resources (DERs), such as photovoltaic systems (PV systems), wind power systems (WPS), electric vehicles (EV) and heat pumps (HP) in distribution networks is constantly increasing, the operation of distribution networks becomes more and more challenging. One of the big challenges of the distribution system operation (DSO) is the potential congestion problems, envisaged as voltage problems (bus voltage is close to or exceeding the limit, typically +/–10%) and overloading problems (loading is close to or exceeding the thermal limit of power components).

The congestions in distribution networks can be caused by simultaneous charging or discharging of EVs, including generation and V2G (vehicle to grid). There are a number of congestion management methods to resolve the problems without reinforcing the distribution network. The congestion management methods can be categorized into two groups, direct control methods [1], [2] and indirect control methods, namely market based methods [3]–[6]. Market based methods have drawn a lot of attention of researchers as well as the DSOs due to the fact that they have least impact on customers and can jointly benefit the stakeholders in the market through energy planning or other operation managements.

As one of the market based methods, the distribution LMP (DLMP) concept [5], [6] has been developed and applied to handle the congestion issues in distribution networks with distributed generators (DGs), by extending the locational marginal price (LMP) concept [7] from transmission networks to distribution networks. The local DGs that produce more power and reduce the energy requirement of the local bus from remote areas during the congestion hours will be properly subsidized under the developed DLMP method.

Another interesting market based method is the dynamic tariff (DT) method [8]–[10], which is derived from the DLMP method. Though the DT method shares many similar features of the DLMP method, e.g. both methods employ optimization tools and marginal cost concept (or shadow price, Lagrange multipliers) and DT is equal to the element of the DLMP corresponding to the congestion cost, there are differences between them. First of all, the DT method is not a market clearing method while the DLMP is. The DT method relies on the existing market, e.g. the spot market in Nordic area, and it can seamlessly integrated into the existing market [8]–[10]. Second of all, the DT method is a decentralized control method, which is implemented through two steps of optimizations, while the DLMP is a centralized clearing method through one step of optimization.

Due to the decentralized control nature of the DT method, the sensitivity analysis of the optimal energy planning to the changes of the parameters of the optimization problem is crucial. The optimal energy planning carried out by the aggregators independently must not be too sensitive to the very small changes of the parameters caused by e.g. numerical errors of the computers. Moreover, it is beneficial for the aggregators to have some level of the freedom of choosing the parameters for their own, e.g. predicted energy prices and/or predicted flexible demands. This paper presents the sensitivity analysis of the dynamic tariff method for congestion management in distribution networks. The intention of the

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study is to obtain the changes of the optimal energy planning and thereby the line loading profiles over the infinitely small changes of parameters.

The paper is organized as follows. The concept of the DT method for congestion management in a decentralized manner and the optimal energy planning through quadratic programing are presented in section II. The sensitivity analysis of the DT method over parameter changes are described in section III. The impact of the parameter changes on the effectiveness of the DT method are shown through three case studies in section IV followed by the conclusion.

II. Quadratic Programming Based Dynmaic Tariff Methods for Congestion Management

A. Concept of Dynamic Tariff for Congestion Management

In [8], [9], the DT method to solve the congestion problem in a decentralized manner is proposed with the following procedure. Firstly, the DSO collects the flexible demand data, such as energy requirements and the availability, from the aggregators or by its own prediction. The DSO also needs the network information and the predicted spot price. Secondly, the DTs are calculated through the optimal energy planning by the DSO where the network constraints are respected. The DTs are sent to all the aggregators, together with the predicted energy prices as the reference to the aggregators. Thirdly, the aggregators make their own optimal plans independently with both the predicted spot prices and the received DTs. At last, the aggregators submit their energy plan/bids to the spot market.

B. Quadratic Programming Based Dynamic Tariff Method

In our previous work [10], a quadratic programing formulation was proposed for the DT method, which can not only take into account the predicted energy price sensitivity of the spot market to the energy planning of the flexible demands, but also offer a unique optimal solution of the energy planning at the aggregator side and therefore resolve the duel degeneracy issues of the optimization problem formulated through linear programming.

Taking EV as an example in this paper, the energy planning of flexible demands at the DSO side can be written as [10],

$$\min_{p_{i,t}} \sum_{i\in N_B, t\in N_T} \frac{1}{2} p_{i,t}{}^T B_{i,t} p_{i,t} + (c_t 1)^T p_{i,t} \ , \tag{1}$$

subject to,

$$\sum_{i\in N_B} DE_i p_{i,t} \le f_t, \forall t\in N_T, (\lambda_t) \ , \tag{2}$$

$$e_i^{\min} \le \sum_{t\_\le t} (p_{i,t\_} - d_{i,t\_}) + e_{i,0} \le e_i^{\max}, \forall \mathrm{t}\in \mathrm{N}_t, i\in N_B \ , \tag{3}$$

$$p_{i,t}^{\min} \le p_{i,t} \le p_{i,t}^{\max}, \forall i\in N_B, t\in N_T \ , \tag{4}$$

Where c_t is baseline price, $d_{i,t} \in R^{m_i}$ is discharging power of EVs due to driving, $e_i^{\min} \in R^{m_i}$ is lower limit of the state of charge (SOC) level, $e_i^{\max} \in R^{m_i}$ is upper limit of the SOC level, $e_{i,0} \in R^{m_i}$ is initial SOC level, $f_t \in R^{n_L}$ is line loading limit available for flexible demands, $p_{i,t} \in R^{m_i}$ is charging power of EVs of one aggregator, $p_{i,t}^{\min} \in R^{m_i}$ is lower charging power limit of EVs, $p_{i,t}^{\max} \in R^{m_i}$ is upper charging power limit of EVs, $B_{i,t} \in R^{m_i\times m_i}$ is matrix of the price sensitivity coefficient, $D\in R^{n_L\times n_d}$ is power transfer distribution factor (PTDF), $E_i \in R^{n_d\times m_i}$ is customer to load bus mapping matrix, $\lambda_t \in R^{n_L}$ is Lagrange multiplier of line loading limit constraint.

The calculated DT through the above optimization by the DSO is $D^T\lambda_t$, which will be sent to the aggregators, who then make the energy planning of the flexible demands on behalf of the owners. The energy planning of different aggregator is independent and without information of the network constraints, i.e. aggregator i can employ the following optimization to make the energy planning [10].

$$\min_{p_{i,t}} \sum_{t\in N_T} \frac{1}{2} p_{i,t}{}^T B_{i,t} p_{i,t} + (c_t 1 + E_i^T D^T \lambda_t)^T p_{i,t} \tag{5}$$

subject to,

$$e_i^{\min} \le \sum_{t\_\le t} (p_{i,t\_} - d_{i,t\_}) + e_{i0} \le e_i^{\max}, \forall \mathrm{t}\in \mathrm{N}_t \tag{6}$$

$$p_{i,t}^{\min} \le p_{i,t} \le p_{i,t}^{\max} \quad \forall t\in N_T \tag{7}$$

III. Sensitivity Analysis of Dynamic Tariff Method

In [9] and [10], the authors have proven the convergence of the aggregator energy planning and the DSO one, i.e. the results and the line loading profiles resulting from the energy planning at the aggregator side will be the same as those at the DSO side, and therefore the DT method for the congestion management is realized in a decentralized manner. An assumption made by the authors is that the parameters of the optimization problem at the DSO side are the same as those of the one at the aggregator side. However, this is not necessarily true. In this section and section IV, the sensitivity of the optimal energy planning results and the line loading profiles to the changes of parameters of the aggregator problem (take the parameters of the DSO problem as the benchmark) will be analyzed.

The sensitivity study shall be done per optimal solution. Assume that the optimal solution of the optimization problem (5)-(7) is $(p_{i,1}^*, p_{i,2}^*, p_{i,3}^*, ..., p_{i,n_T}^*)$. Under the assumption that the changes of the parameters are infinitely small, only the constraints that are 'active' (equality is hold) at the optimal point will contribute to the sensitivity study. The optimization problem can be rewritten as below (i is fixed, since the aggregators make the energy planning separately and the sensitivity analysis can be performed separately for each aggregator),

$$\min_x \quad \frac{1}{2} x^T Bx + g^T x \tag{8}$$

subject to,

$$Ax = b, (\mu) \tag{9}$$

where x is $\left[p_{i,1}{}^T, p_{i,2}{}^T, p_{i,3}{}^T, ..., p_{i,n_T}{}^T \right]^T$,

B is $\begin{bmatrix} B_{i,1} & & & & \\ & B_{i,2} & & & \\ & & B_{i,3} & & \\ & & & ... & \\ & & & & B_{i,n_T} \end{bmatrix}$,

g is $\begin{bmatrix} c_1 1 + E_i^T D^T \lambda_1 \\ c_2 1 + E_i^T D^T \lambda_2 \\ c_3 1 + E_i^T D^T \lambda_3 \\ ... \\ c_{n_T} 1 + E_i^T D^T \lambda_{n_T} \end{bmatrix}^T$,

A and b are coefficients of the active constraints in (6) and (7), μ is Lagrange multiplier of the active constraints.

According to the KKT conditions that the optimal solution must fulfill, the following nonlinear equations can be obtained.

$$Bx + g - A^T \mu = 0 \tag{10}$$

$$Ax - b = 0 \tag{11}$$

Using implicit differentiation theory or Solving the above equations and using normal differentiation theory [11], the following partial differentiation can be achieved, which indicates how the optimal solution changes over the changes of the energy prices.

$$\frac{\partial x}{\partial c} = \frac{\partial x}{\partial g} = B^{-1} A^T (AB^{-1} A^T)^{-1} AB^{-1} - B^{-1} \tag{12}$$

where c is $\left[c_1{}^T, c_2{}^T, c_3{}^T, ..., c_{n_T}{}^T \right]^T$.

The line loading change at a particular hour t is the summation of the changes of the EV charging power, which can be shown as ,

$$\sum_{i \in N_B} DE_i (\Delta p_{i,t}) = \sum_{i \in N_B, t\_ \in N_T} DE_i \frac{\partial p_{i,t}}{\partial c_{t\_}} \Delta c_{t\_} \tag{13}$$

where $\frac{\partial p_{i,t}}{\partial c_{t\_}}$ can be retrieved from matrix $\frac{\partial x}{\partial c}$ by taking the corresponding rows and columns.

The sensitivity to the changes of other parameters, such as price sensitivity coefficient ($B_{i,t}$), the energy requirement ($d_{i,t}$, $e_i^{\min}$, $e_i^{\max}$, $e_{i,0}$) and EV availability, can be obtained through similar procedures.

IV. CASE STUDIES

Case studies were conducted using the Danish driving pattern and the Bus 4 distribution system of the Roy Billinton Test System (RBTS) [12] to demonstrate the impact of small and big changes of parameters on the line loading profiles and the effectiveness of the DT method under parameter changes. The details of the case studies are presented in this section.

A. *Grid Data*

The single line diagram of the Bus 4 distribution network is shown in Fig. 1. Line segments of the feeder one are labeled in Fig. 1, among which L2, L4, L6, L8, L9, L11, and L12 refer to the transformers connecting the corresponding load points (LP1 to LP7). The study is focused on this feeder because it has the most diversity among all the feeders: 5 residential load points with different peak conventional demands and two commercial load points. Each of the residential load points (LP1-5) has 200 customers while each of the commercial load points (LP6-7) has 10 customers. The peak conventional demands of residential customers are assumed to occur at 18:00 when people come home and start cooking (shown in Fig. 4).

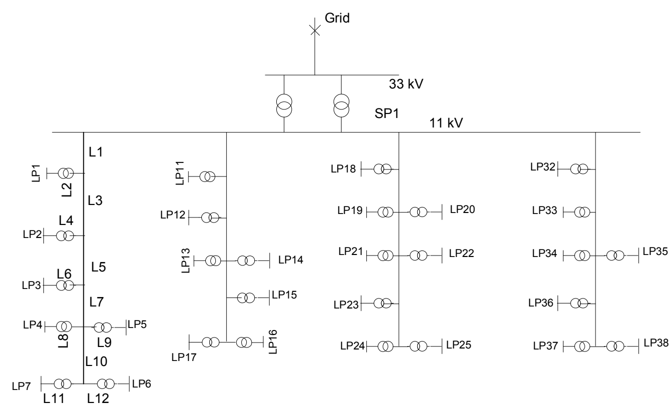

Fig. 1. Single line diagram of the distribution network

The key parameters of the simulations are listed in Table I. The EV availability shown in Fig. 2 is from the driving pattern study in [13]. The system prices shown in Fig. 3 are used as the benchmark energy prices. Assume that there are two aggregators (agg1 and agg2); one has 40 customers per load point and the other has 160 customers per load point.

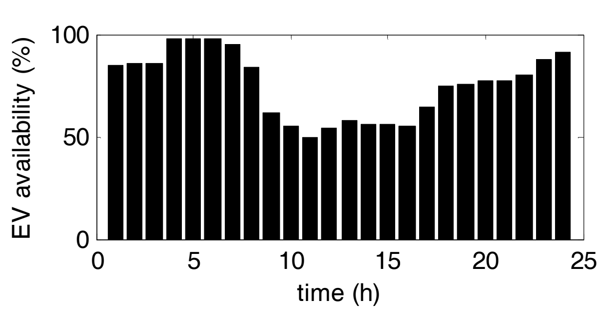

Fig. 2. EV availability

TABLE I
KEY PARAMETERS FOR THE SIMULATION

Parameter	value
EV battery size	25 kWh
Peak charging power	11 kW (3 phase)
Energy consumption per km	150 Wh/km
Minimum SOC	20%
Maximum SOC	85%
Average driving distance	40 km
Line loading limit: L2	1400 kW
Line loading limit: L3	6000 kW
Line loading limit: L4	1700 kW

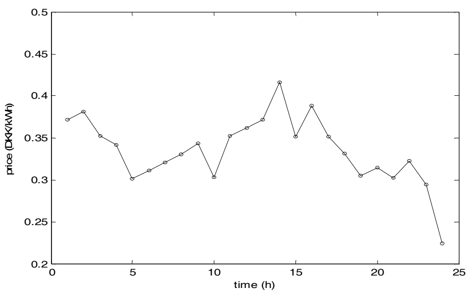

Fig. 3. System prices (day-ahead spot prices)

B. *Results of The Sensitivity Analysis*

According to the framework of the DT method for congestion management, the DSO will firstly calculate the DT through an energy planning of the flexible demands (EVs) in the whole distribution network based on the prediction of the energy prices and the energy requirements of the flexible demands. The energy planning results are shown in Fig. 4. It can be seen that the line loading limits are respected at DSO side energy planning.

The calculated DTs are sent to the aggregators, who independently make energy plans based on the received DTs and predicted energy prices. In order to compare with the sensitivity study of different cases, the base case is firstly presented, where the predicted energy prices at the aggregator side are the same as those at the DSO side. As expected, the line loadings resulting from the energy planning at the aggregator side (see Fig. 5) are the same as those at the DSO side.

1) *Case One:*

In Case One, the predicted energy prices at the aggregator side are 5% higher than those at the DSO side, i.e. the relative error is 5% (take the one at the DSO side as the benchmark). The line loadings resulting from the energy planning at the aggregator side are shown in Fig. 6. Due to the relative error, the energy planning at the aggregator side is no longer same as the one at the DSO side, leading to the fact that the over loadings are not completely alleviated by the DT method. The overloading of L3 at hour 24 is about 5.1%, because the DLMPs at hour 24 become more attractive than the other hours under the assumption of 5% relative error of predicted energy prices.

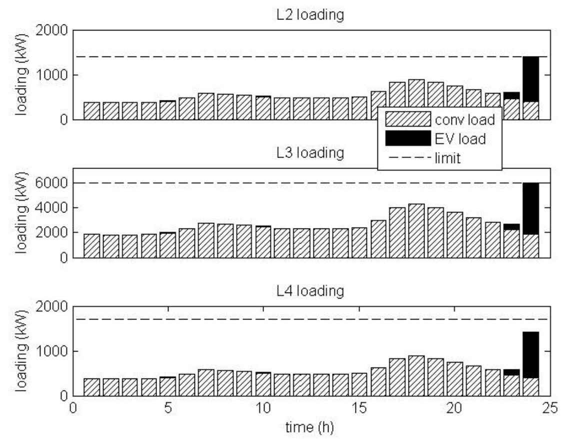

Fig. 4. Line loading of the energy planning at DSO side

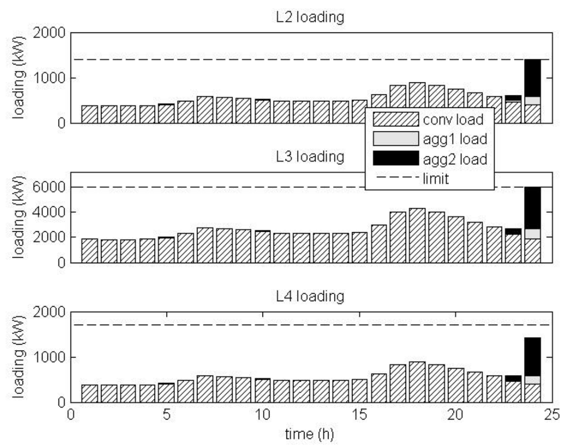

Fig. 5 Line loading of the energy planning at aggregator side (base case)

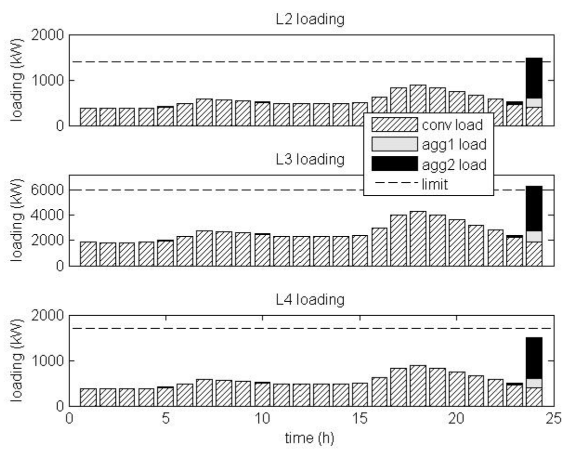

Fig. 6 Line loading of the energy planning at aggregator side (Case One)

2) *Case Two:*

In Case Two, the predicted energy prices at the aggregator side are 5% lower than those at the DSO side, i.e. the relative error is -5%. The aggregators use such energy prices with negative relative error leading to a better line loading profile. The results can be seen in Fig. 7. Headroom about 5.2% between the line loading of L3 and the limit is observed at hour 24.

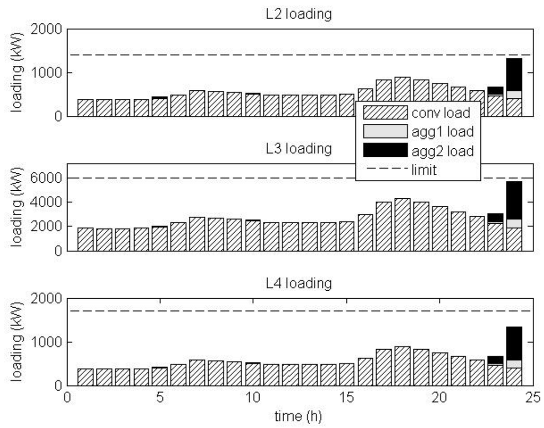

Fig. 7 Line loading of the energy planning at aggregator side (Case Two)

The results of Case One and Case Two studies agree with the results of the sensitivity analysis on infinite small changes through formulas (12) - (13), which are tabularized in Table II (only the results regarding L3 are shown). For instance due to the fact that the absolute change of the energy price at hour 23 is bigger than hour 24, when the energy price prediction has positive errors (Case One) the total effect on line loading change at hour 24 is positive (see Fig. 6).

TABLE II
SENSITIVITY MATRIX OF L3 LOADING CHANGE TO ENERGY PRICE CHANGE, DATA OF OTHER HOURS IS ZERO (UNIT: MW/(DKK/KWH))

HOUR	**5**	**6**	**10**	**19**	**23**	**24**
5	-8	2	4	2	0	0
6	2	-6	2	2	0	0
10	4	2	-8	2	0	0
19	2	2	2	-6	0	0
23	0	0	0	0	-88	88
24	0	0	0	0	88	-88

The study of Case One and Two assumes that the aggregators have the same relative error of the energy price prediction. It can be expected that if they have different relative error, e.g. one has positive error while the other has negative error, the results of the loading profile will between those of Case One and Two.

3) Case Three:

The third case study is based on the assumption that the maximum DT that can be employed by the DSO is preset by regulation laws in order to prevent the abuse of DT by the DSO. One can anticipate how serious the impact of such DT limitation on the DT method for congestion management can be through the sensitivity analysis presented in section III, because the DT ($D^T\lambda_t$) is subject to some changes due to the limitations, though the assumption of the infinitely small changes is not necessarily held.

The actual impact of the DT limitation is illustrated through simulations. The results of a simulation are shown in Fig. 8, where the DT limitation is set to be 20% of the maximum energy price difference of different hours, e.g. hour 14 and 24 in Fig. 3. It can be seen from Fig. 8 that the effectiveness of the DT method for congestion management is completely lost due to such a strong DT limitation (the EV charging power should be shifted a bit to hour 23 from hour 24 if the DT method has effect on the congestion management in this case).

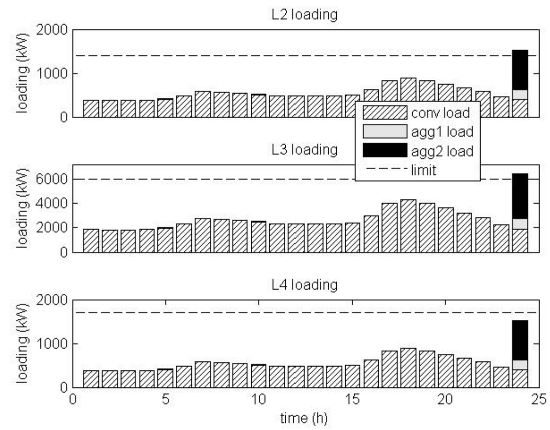

Fig. 8 Line loading of the energy planning at aggregator side (Case Three)

V. CONCLUSIONS

The paper presents the sensitivity analysis of the DT method for congestion management in distribution networks which obtains the optimal energy planning and thereby the line loading profiles with respect to the changes of the parameters of the optimization problem at the aggregator side. The change of the optimal energy planning over infinitely small changes of the parameters can be calculated through explicit formulas for each optimal point. Three case studies demonstrate that small changes of the predicted energy prices (e.g. +/-5% relative error) lead to an acceptable line loading profiles with some tolerance (e.g. +/-5%). Big changes of the DT due to strong DT limitations can fail the DT method for congestion management.

REFERENCES

[1] A. R. Malekpour, A. R. Seifi, M. R. Hesamzadeh, and N. Hosseinzadeh, "An optimal load shedding approach for distribution networks with DGs considering capacity deficiency modelling of bulked power supply," in *Proc. Power Engineering Conference*, pp. 1–7.

[2] M. Alonso, H. Amarís, C. Álvarez, and R. Albarracín, "Reactive power planning in distribution networks with distributed generation," in *Proc. 7th Mediterranean Conference and Exhibition on Power Generation, Transmission, Distribution and Energy Conversion*, pp. 152–152.

[3] R. A. Verzijlbergh, L. J. De Vries, and Z. Lukszo, "Renewable energy sources and responsive demand. Do we need congestion management in the distribution grid?," *IEEE Trans. Power Syst.*, pp. 1–10, 2014.

[4] P. Bach Andersen, J. Hu, and K. Heussen, "Coordination strategies for distribution grid congestion management in a multi-actor, multi-objective setting," in *Proc. 3rd IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*, pp. 1–8.

[5] P. M. Sotkiewicz and J. M. Vignolo, "Nodal pricing for distribution networks: efficient pricing for efficiency enhancing DG," *IEEE Trans. Power Syst.*, vol.21, no.2, pp. 1013–1014, May 2006.

[6] G. T. Heydt, B. H. Chowdhury, M. L. Crow, D. Haughton, B. D. Kiefer, F. Meng, and B. R. Sathyanarayana, "Pricing and control in the next generation power distribution system," *IEEE Trans. Smart Grid*, vol.3, no.2, pp. 907–914, Jun. 2012.

[7] R. Bohn, M. Caramanis, and F. Schweppe, “Optimal pricing in electrical networks over space and time,” *RAND J. Econ.*, vol.15, no.3, pp. 360–376, 1984.

[8] N. O’Connell, Q. Wu, J. Østergaard, A. H. Nielsen, S. T. Cha, and Y. Ding, “Day-ahead tariffs for the alleviation of distribution grid congestion from electric vehicles,” *Electr. Power Syst. Res.*, vol.92, pp. 106–114, 2012.

[9] R. Li, Q. Wu, and S. S. Oren, “Distribution locational marginal pricing for optimal electric vehicle charging management,” *IEEE Trans. Power Syst.*, vol.29, no.1, pp. 203–211, Jan. 2014.

[10] S. Huang, Q. Wu, S. S. Oren, R. Li, and Z. Liu, “Distribution locational marginal pricing through quadratic programming for congestion management in distribution networks,” *IEEE Trans. Power Syst.*, in Press.

[11] J. C. G. Boot, “On Sensitivity Analysis in Convex Quadratic Programming Problems,” *Oper. Res.*, vol.11, no.5, pp. 771–786, Oct. 1963.

[12] R. N. Allan, R. Billinton, I. Sjarief, L. Goel, and K. S. So, “A reliability test system for educational purposes-basic distribution system data and results,” *IEEE Trans. Power Syst.*, vol.6, no.2, pp. 813–820, May 1991.

[13] Q. Wu, A. H. Nielsen, J. Østergaard, F. Marra, and C. Træholt, “Driving pattern analysis for electric vehicle (EV) grid integration study,” in *Proc. 2010 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT Europe)*, pp. 1–6.